



MATHEMATICS
STANDARD LEVEL
PAPER 1

Wednesday 5 May 2010 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



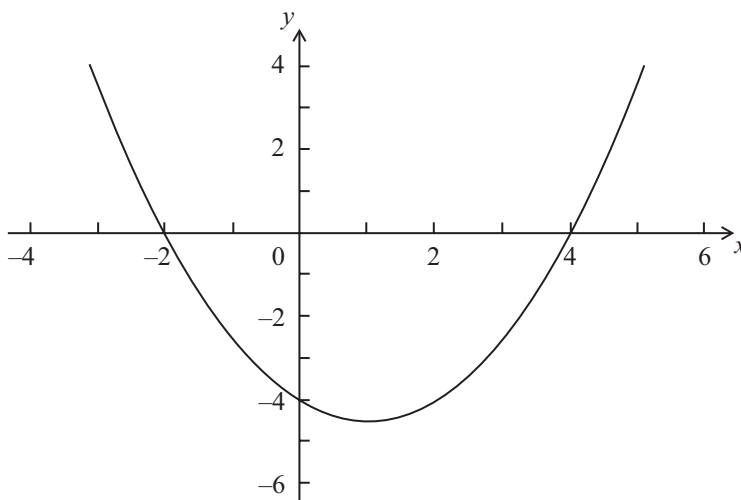
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Let $f(x) = p(x - q)(x - r)$. Part of the graph of f is shown below.



The graph passes through the points $(-2, 0)$, $(0, -4)$ and $(4, 0)$.

- (a) Write down the value of q and of r . [2 marks]
- (b) Write down the **equation** of the axis of symmetry. [1 mark]
- (c) Find the value of p . [3 marks]

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2. [Maximum mark: 8]

Let $\vec{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\vec{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

(a) Find \vec{BC} . [2 marks]

(b) Find a unit vector in the direction of \vec{AB} . [3 marks]

(c) Show that \vec{AB} is perpendicular to \vec{AC} . [3 marks]

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3. [Maximum mark: 5]

Let $A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$.

(a) Find AB .

[3 marks]

(b) Solve $A^{-1}X = B$.

[2 marks]

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4. [Maximum mark: 7]

Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

(a) Find $f\left(\frac{\pi}{2}\right)$. [2 marks]

(b) Find $(g \circ f)\left(\frac{\pi}{2}\right)$. [2 marks]

(c) Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of k , $k \in \mathbb{Z}$. [3 marks]

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5. [Maximum mark: 6]

Let $f(x) = kx^4$. The point $P(1, k)$ lies on the curve of f . At P , the normal to the curve is parallel to $y = -\frac{1}{8}x$. Find the value of k .

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6. [Maximum mark: 7]

Solve $\log_2 x + \log_2 (x - 2) = 3$, for $x > 2$.

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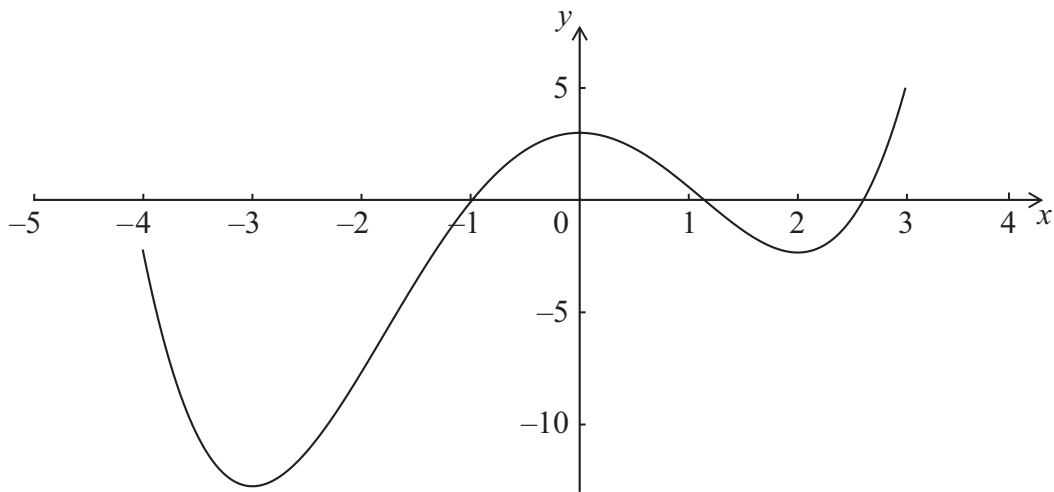
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7. [Maximum mark: 6]

A function f is defined for $-4 \leq x \leq 3$. The graph of f is given below.



The graph has a local maximum when $x = 0$, and local minima when $x = -3, x = 2$.

- (a) Write down the x -intercepts of the graph of the **derivative** function, f' . [2 marks]
- (b) Write down all values of x for which $f'(x)$ is positive. [2 marks]
- (c) At point D on the graph of f , the x -coordinate is -0.5 . Explain why $f''(x) < 0$ at D. [2 marks]

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SECTION B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 14]

Consider the function f with second derivative $f''(x) = 3x - 1$. The graph of f has a minimum point at $A(2, 4)$ and a maximum point at $B\left(-\frac{4}{3}, \frac{358}{27}\right)$.

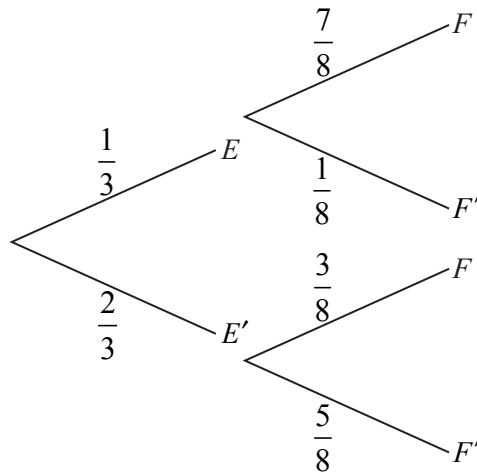
- (a) Use the second derivative to justify that B is a maximum. [3 marks]
- (b) Given that $f'(x) = \frac{3}{2}x^2 - x + p$, show that $p = -4$. [4 marks]
- (c) Find $f(x)$. [7 marks]



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9. [Maximum mark: 14]

José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.
 If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.
 If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.
 Let E be the event “he misses his bus” and F the event “he is late for school”.
 The information above is shown on the following tree diagram.



(a) Find

(i) $P(E \cap F)$;

(ii) $P(F)$.

[4 marks]

(b) Find the probability that

(i) José misses his bus and is not late for school;

(ii) José missed his bus, given that he is late for school.

[5 marks]

The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday.

(c) **Copy** and complete the probability distribution table.

[3 marks]

X (cost in euros)	0	3	6
$P(X)$	$\frac{1}{9}$		

(d) Find the expected cost for José for both days.

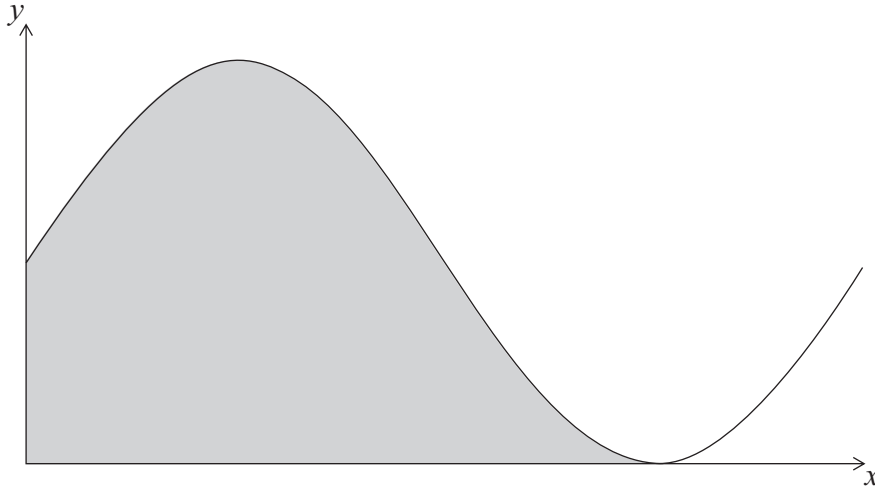
[2 marks]



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10. [Maximum mark: 17]

Let $f(x) = 6 + 6\sin x$. Part of the graph of f is shown below.



The shaded region is enclosed by the curve of f , the x -axis, and the y -axis.

- (a) Solve for $0 \leq x < 2\pi$
 - (i) $6 + 6\sin x = 6$;
 - (ii) $6 + 6\sin x = 0$. [5 marks]
- (b) Write down the exact value of the x -intercept of f , for $0 \leq x < 2\pi$. [1 mark]
- (c) The area of the shaded region is k . Find the value of k , giving your answer in terms of π . [6 marks]

Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of f is transformed to the graph of g .

- (d) Give a full geometric description of this transformation. [2 marks]
- (e) Given that $\int_p^{p+\frac{3\pi}{2}} g(x) dx = k$ and $0 \leq p < 2\pi$, write down the two values of p . [3 marks]

